Tolerance requirements revisited for the calibration of chaotic reverberation chambers

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Abstract. This paper aims to revisit the tolerance requirements for the calibration of chaotic reverberation chambers (RC). In recent years, some of the authors have established how chaotic reverberation chambers can ideally meet the requirement of statistically isotropic, uniform and depolarized response fields due to so-called ergodic modes which are generic in chaotic cavities. In particular, it was shown that, below or close to the *lowest useable frequency* (LUF), a generally admitted criterion proposed by the International Standard IEC 61000-4-21 to evaluate the uniformity of the field distribution is valid in chaotic RCs while not in conventional regular RCs. Indeed, the EM field in chaotic RCs display universal statistical features which cannot be obtained in conventional RCs at low frequency. In chaotic RCs, we propose to use the universality of field statistics to evaluate a confidence interval based on the theoretical distribution of the electric field deduced from the sole knowledge of the modal overlap.

I. Introduction

Electromagnetic (EM) reverberation chambers are commonly used for electromagnetic compatibility (EMC) tests [1]. Thanks to mechanical or electronic stirring and to the modal overlap resulting from loss mechanisms, the resulting field is assumed to be statistically isotropic, uniform and depolarized. Such properties correspond to the *so-called* Hill's hypotheses [2] when the excitation frequency is well above the LUF [1]. In this regime, according to such assumptions, the EM field can be seen as a random superposition of travelling plane waves. Below or close to the LUF, where modal overlap is small or moderate, individual modes contribute to the EM response. In a conventional RC of rectangular shape, modes with simple spatial structure are responsible for highly non-isotropic patterns yielding a response which cannot be expected to be statistically isotropic and uniform, despite the presence of stirring. To the contrary, these statistical features are naturally displayed by the vast majority of modes in a chaotic cavity without the help of any stirring process [3]. This generic statistical behavior of modes in chaotic cavities will be coined *ergodicity* hereafter. By comparing the experimental EM responses in a chaotic RC and in a conventional mode-stirred RC, the ergodicity of modes in a chaotic RC have been demonstrated to play a key role in improving the statistical behavior of RCs in the neighborhood of the LUF [4]–[7].

This paper considers the uniformity criterion of International Standard IEC 61000-4-21 [1] applied to the calibration of chaotic RCs such as those which have been studied in [6] and [7]. According to [1], the fluctuations of the maxima of the field amplitude can be evaluated through:

$$\sigma_{dB} = 20\log_{10}\left(1 + \frac{\sigma_{\max}}{\langle |E_a|_{\max}\rangle_8}\right) \tag{1}$$

where 8 measurement points are chosen in the RC. At each of these locations, for N uncorrelated positions of the stirrer, one extracts the amplitude of the field component $|E_a|$ and one keeps the maximum value $|E_a|_{\max}$. One then computes the average and the standard deviation over the 8 values of $|E_a|_{\max}$.

In the following section, the theoretical statistical distribution of the electric field of a chaotic RC deduced from a Random Matrix model introduced in [5] and [7] will be discussed. Then we will estimate the expected value of σ_{dB} with quantiles at 2.5% and 97.5% of the distribution of σ_{dB} deduced from a Monte-Carlo simulation based on the validated model.

II. Statistics of the response field of a chaotic cavity

Contrary to Hill's assumptions, below or close to the LUF, the real and imaginary parts of each component of the field are not identically distributed [4]. For a given configuration of an ideally chaotic cavity (or a given frequency, relying on ergodicity), they still are independently Gaussian distributed, but with different variances [4]. The ensuing distribution of the modulus of each component $|E_a|$ is then no longer a Rayleigh distribution [12] but depends on a single parameter ρ, called the *phase rigidity*, defined by:

$$\rho = \frac{\int_V \vec{E}\cdot\vec{E}\, d\vec{r}}{\int_V ||\vec{E}||^2\, d\vec{r}}\,. \tag{2}$$

Note that for $|\rho| \to 1$ the system tends to be closed, a situation corresponding to non-overlapping resonances, whereas $|\rho| \to 0$ corresponds to a completely open system, which is the limit of Hill's assumptions. More specifically, due to the *ergodicity* of the modes contributing to the response, for a given excitation frequency and a given configuration, the probability distribution of the normalized field amplitude of the Cartesian component $\mathcal{E}_a = |E_a| / \langle |E_a|^2 \rangle_{\vec{r}}^{1/2}$ depends solely on the modulus of ρ [5], [7] and is given by

$$P(\mathcal{E}_a;\rho) = \frac{2\mathcal{E}_a}{\sqrt{1-|\rho|^2}} \exp\left[-\frac{\mathcal{E}_a^2}{1-|\rho|^2}\right] \mathrm{I}_0 \left[\frac{|\rho|\,\mathcal{E}_a^2}{1-|\rho|^2}\right] . \tag{3}$$

Since the *phase rigidity* is itself a distributed quantity, the distribution of the normalized field amplitude for an ensemble of responses resulting from stirring reads

$$P_a(\mathcal{E}_a) = \int_0^1 P_\rho(\rho) P(\mathcal{E}_a;\rho) d\rho \tag{4}$$

where P_ρ is the distribution of the *phase rigidity* of the responses.

Preliminary investigations, based on numerical simulations of the Random Matrix model described in [4], [5], show that P_ρ depends only on the mean modal overlap d as long as this parameter remains weak or moderate. This overlap d is defined as the ratio of the average modal bandwidth to the average difference between neighbouring modal frequencies. We assume that within the investigated frequency range, the latter quantities vary only slightly. An Antsatz was proposed in [7] to determine P_ρ from the only knowledge of the mean modal overlap, which can be in turn related to the frequency of operation and the composite Q-factor of the cavity through $d \simeq 8\pi V f^3 c^{-3} Q^{-1}$ where V is the volume of the cavity. This Ansatz reads:

$$P_\rho^W(\rho) = \frac{2B \exp[-2B\rho/(1-\rho)]}{(1-\rho)^2} , \tag{5}$$

where the parameter B has a smooth $d-$dependence [7] numerically deduced from our Random Matrix model presented in [5]. An empirical estimation of $B(d)$ is given by

$$B(d) = \frac{ad^2}{1+bd} . \tag{6}$$

where $a = 0.58 \pm 0.04$ and $b = 2.3 \pm 0.3$ [7].

The RC we investigated has a volume $V \simeq 19$ m^3 and was made chaotic by the addition of three or six metallic half-spheres with a radius of 40 cm located on different walls (see Fig. 1 and [6], [7].). The commercial RC we have modified consists in a rectangular metallic cavity of dimensions W =2.95 m, L =2.75 m and H =2.35 m equipped by a mode-stirrer in rotation around the vertical axis and located in a corner. In both configurations, the S-matrix was measured between two antennas (one dipole fixed at a wall and one monopole fixed on a moveable tripod) located at $\vec{r}_1$ and $\vec{r}_2$. Measurements

Fig. 1. 180-degree photograph of the chaotic RC (the commercial RC we modified consists in a rectangular metallic cavity of dimensions W =2.95 m, L =2.75 m and H =2.35 m). Three hemispheres with a radius of 40 cm (one at the ceiling and two on adjacent walls) and a mechanical stirrer in the far corner can be seen.

were realized in the configuration with 3 metallic half-spheres for 1024 regularly spaced frequencies in the interval $[390\text{ MHz}, 410\text{ MHz}]$, for 30 positions of the stirrer spaced by 12 degrees and for 8 different positions of the monopole antenna inside the volume. Once the coupling strength of the antennas was extracted [4], [10], [11], one could deduce from the measurement of S_{12} the normalized value of the amplitude of the Cartesian component of the field along the monopole antenna:

$$\vec{E}(\vec{r}_2, f) \cdot \hat{n}_a = \frac{S_{12}(f)}{\kappa_1 \kappa_2} . \tag{7}$$

where κ_i $(i = 1, 2)$ are the stirring-averaged values of the frequency-dependent coupling constants of the antennas, and $\hat{n}_a$ is the unit vector along the polarisation of antenna 2. In the above cited frequency range, the mean quality factor was estimated to be around 1500 corresponding to a mean modal overlap $d{\sim}0.89$, which was deduced from the measurements by extracting the complex resonances using the method of harmonic inversion [4], [7].

As an illustration, Fig. 2 shows a comparison of the histogram of the normalized field amplitude built from experimental data, with the prediction (4) where the value $d = 0.89$ was used. A best fit with a Weibull distribution used in [12] is also shown, clearly indicating that the latter cannot account for the experimental data neither in the bulk of the distribution nor on the tails.

III. Tolerance requirements for a chaotic cavity

Using the above distribution for the normalized field amplitude and relying on the ergodic hypothesis for the response, we performed a Monte-Carlo simulation using Eq. (4) with 4096 estimations of σ_{dB}. Each simulation provides one estimation of σ_{dB}, from 8 estimations of the maximum of N values. Three different values of d were used for the Monte-Carlo simulations: d =0.9, 0.4, and 0.2. The two higher values correspond to two frequency ranges of the chaotic RC investigated in [6], [7] and [8], namely [390 MHZ, 410 MHz] and [220 MHZ, 270 MHz], respectively. The results for the confidence interval (95%)

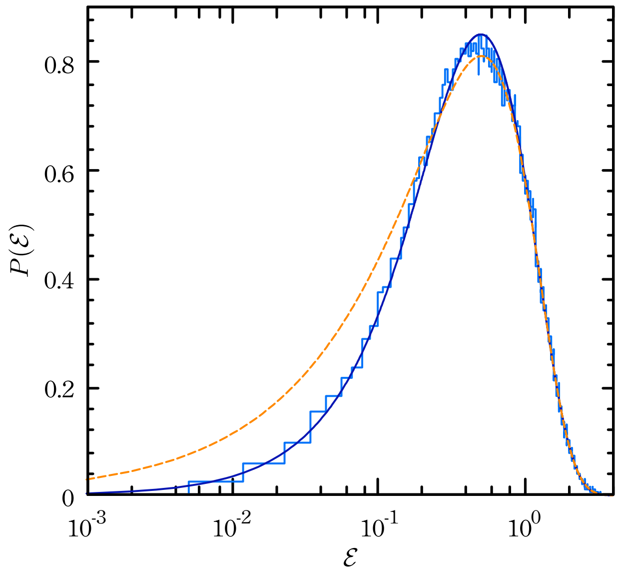

Fig. 2. Histogram of the normalized field component amplitude from experimental data, compared with the prediction (4) (continuous line) with the corresponding probability distribution of Eq.(4) where the value $d = 0.89$ was used. Weibull best fit (dashed curve).

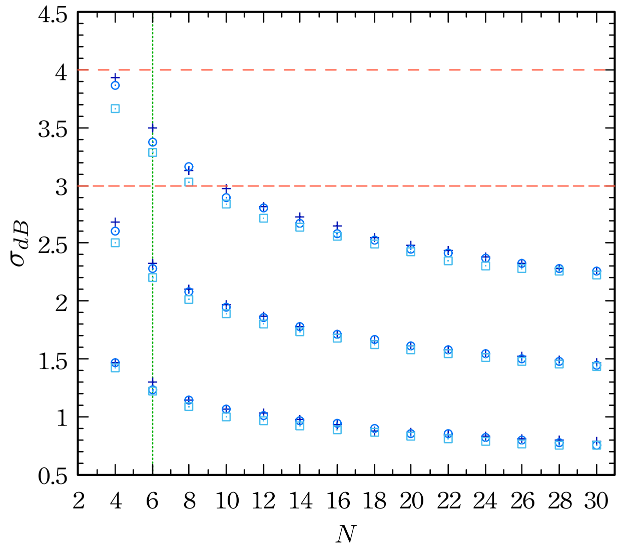

Fig. 3. Confidence interval (95%) of σ_{dB}, deduced from Monte-Carlo simulation. N is the sample size from which the maximum is selected. It corresponds to the number of uncorrelated configurations corresponding e.g. to a possibly larger number of stirrer positions. The expected value and the quantiles 2.5% and 97.5% are shown for 3 values of the modal overlap $d = 0.9$ ($\boxdot$), $d = 0.4$ ($\odot$), $d = 0.2$ ($+$). The horizontal dashed lines correspond to the thresholds 3 and 4 dB. The vertical dashed line indicates the effective number of uncorrelated configurations corresponding to our experimental results in the frequency band [390 MHZ, 410 MHz].

of σ_{dB} are shown in Fig. 3. As the effective number N of uncorrelated realizations is increased, the expected values and the quantiles seem to coincide irrespective of the modal overlap. However, for values of N smaller than 10, significant deviations of expected values and quantiles can be seen for the three values of the modal overlap which are shown. The standard IEC 61000-4-21 defines a threshold of 3 dB above the LUF increasing to 4 dB at lower frequencies [1], [9]. Thus, with a confidence interval of 95%, σ_{dB} does not exceed 4 dB in a chaotic cavity as long as the effective sample size exceeds $N = 4$. To attain a 3 dB threshold, it has to reach a number of uncorrelated realizations $N = 10$. Note that the mean values for both cases are 2.5 and 1.9, respectively.

IV. Experimental and numerical validation

In our experiments, for a frequency around 400 MHz, corresponding to a modal overlap close to $d \simeq 0.9$, we used 30 positions of the rotating stirrer spaced by 12 degrees. A mean value $\langle\sigma_{dB}\rangle = 2.18$ was experimentally and numerically found, which agrees with our present results if one assumes an effective number of uncorrelated stirrer positions $N \simeq 6$. This is indeed substantiated by the correlation coefficient between tuner steps we calculated from our experimental data. It is presented in Fig. 4 where 0.37 is reached after roughly 5 steps. The results of our simulation lead to $N \simeq 6$ as the effective number of uncorrelated stirrer positions, to yield the experimentally observed mean value of σ_{dB}. This result is fully consistent with the fact that among 30 regularly spaced steps of the stirrer, only 6 positions can be considered as independent every 5 steps. Note that the more measurement points (here 8) we take into account to estimate the maximum values, the more the confidence interval of σ_{dB} is reduced, as a consequence of the ergodicity of the response of chaotic cavities. Additionally, the validity of a Monte-Carlo simulation essentially relies on the ergodicity of the system, a feature which indeed holds for chaotic cavities at or even below the LUF whereas it is not verified in a regular conventional mode-stirred RC at low frequencies due to non-universal statistics of modes of regular cavities. Indeed, in the commercial regular RC we used, without any hemisphere, in the same frequency range, we have previously shown numerically that the expected value of $\langle\sigma_{dB}\rangle$ significantly vary with the number of measurement points and displays no convergence when this number is increased [6]. Moreover, for 8, 16, and 64 measurement points in the cavity, numerical simulations of the response in the frequency band [390 MHZ, 410 MHz] yielded percentages of σ_{dB} values above 3 dB equal to 25.7 %,16 % et 27.3 %, respectively.

Thus, from the universal properties of chaotic cavities and the knowledge of the modal overlap d and of the number of uncorrelated configurations N, which can be easily deduced from measurements, we have established it is possible to provide a well-founded confidence interval of σ_{dB}.

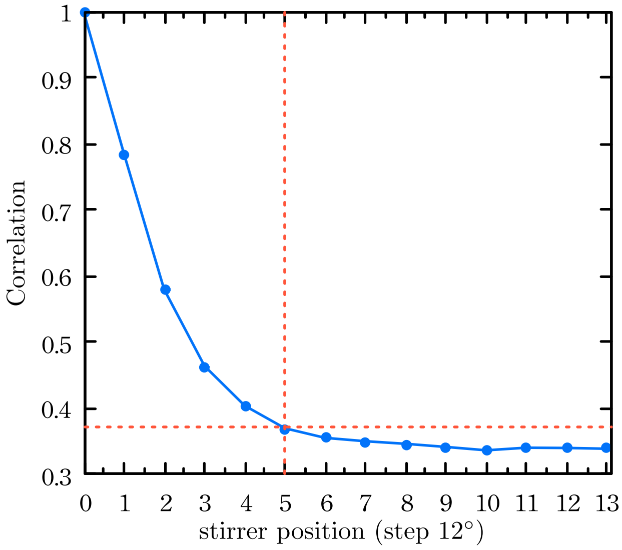

Fig. 4. Correlation coefficient between tuner steps for a frequency around 400 MHz. The value 0.37 corresponds to 5 steps, in perfect agreement with an effective number of uncorrelated stirrer positions $N \simeq 6$ out of the 30 steps for a complete rotation of the stirrer.

V. Conclusion

This article presents an investigation of the tolerance requirements for the calibration of chaotic reverberation chambers. By means of a theoretical approach based on the random matrix theory applied to chaotic open wave systems, we are able to predict the distribution of any normalized electric field component of the response in terms of a unique parameter, namely the mean modal overlap. This distribution is successfully used to evaluate a confidence interval through a Monte-Carlo simulation, in complete agreement with experimental results obtained in a chaotic reverberation chamber. Our results confirm the key role of the ergodic character of the response of a chaotic reverberation chamber to improve the statistical behavior of a reverberation chamber in the neighborhood of the LUF. In particular, these results demonstrate that the criterion proposed by the standard to evaluate the uniformity of the spatial field distribution, when used near the LUF or at lower frequencies, is only valid if the reverberation chamber is chaotic since the *ergodicity* of the response cannot be guaranteed in the case of a regular reverberation chamber.

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